General Equilibrium and the Emergence of (Non) Market Clearing Trading Institutions

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Introduction

- Motivation
  - Market institution
    i.e. trading rules that determine the matching and price formation process.
  - Market institutions matter
    for efficiency, surplus distribution, convergence to market clearing outcome (Plott 1982, Holt 1993, Ausubel and Cranton 2002, Ockenfels and Roth 2002).

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- Claim (Hayek etc.):
  - *Because of efficiency reasons, only trading institutions that guarantee market clearing survive in the long run.*
- Questions:
  - Is there any mechanism that guarantees that existing market institutions support market-clearing outcomes?
  - If several trading institutions exist, which one survives in the long run?
  - If traders have to choose between different trading institutions, will they learn to choose a market-clearing (efficient) one?

Does Learning lead to Market-Clearing?

- **Buyers-Sellers Model:** Alós-Ferrer & Kirchsteiger, 2004.
  - Finite number of potentially biased institutions for trading a single homogeneous good.
  - Institutions exogenously given
  - Bias: price above or below market-clearing one, implies rationing of long-side.
  - Finite number of buyers and sellers; types exogenous.
  - Myopic behavior: move to institutions currently perceived as best.

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  - Finite number of buyers and sellers; types exogenous.
  - Myopic behavior: move to institutions currently perceived as best.
- **Results:**
  - First: In the long run, a market-clearing institution always survives.
  - Why? Key Result: When comparing a market-clearing institution with a non-market clearing one, always either buyers or sellers are better off in the market-clearing one.
  - Second: Depending on the details of the dynamics, non-market clearing institutions might also survive in the long run.
  - Why? Not always both types of traders are better off in the non-market clearing institution.

Overview of this paper

- General Equilibrium framework.
- Pure exchange economy with finitely many traders (not constrained to be buyers or sellers).
- Finitely many goods.
- Finitely many institutions per good, one of them market-clearing.
- The others exhibit price bias and rationing.
- Traders are boundedly rational in their choice of institutions, focusing on perceived good results.
- Will traders learn to coordinate on the various market-clearing institutions?

The Model

The Exchange Economy

- $i = 1, ..., N$ traders, $k = 1, ..., K$ commodities, plus a *numeràire* $k = 0$.
- Each trader is characterized by excess demand functions $x_k^i\left(p^i\right)$ where $p^i \in \mathbb{R}_+^K$ is the price vector with which trader i is confronted; prices are measured in units of the *numeràire* good.

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We consider a very regular economy.
(A0) For all $i = 1, \ldots, N$, $k = 1, \ldots, K$, excess demand functions fulfill

(i) $x_k^i\left(p^i\right)$ is differentiable and strictly decreasing in p_k^i;

(ii) there exists an $a > 0$ such that for all $p^i \in \mathbb{R}_+^K$, $x_k^i\left(p^i\right) > -a$;
No trader can sell short.

(iii) there exists a $p^i \in \mathbb{R}_+^K$, such that $x_k^i\left(p^i\right) < 0$;
Implies positive endowments of every good.

(iv) if $p^{in} \to p^i$ where $p^i \neq 0$ and $p_k^i = 0$, then $x_k^i\left(p^{in}\right) \to \infty$;
Fulfilled with strongly monotone preferences.

(v) for all $k \neq l$, $\frac{\partial x_k^i\left(p^i\right)}{\partial p_l} > 0$. Goods are gross substitutes.

Choice of Institutions

- Viewed as a (coordination) game.
- For each good $k \neq 0$ there exists a finite, nonempty set Z_k of institutions at which this good can be traded.
- Each period, each trader decides for each good the institution at which he wants to trade.
- The strategy set of a every trader i is given by

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- Given a strategy profile $s \in S = \prod_{i=1}^{N} S^i$, denote by $N(s, z)$ the set of players who have chosen to trade good $k \neq 0$ at institution $z \in Z_k$.
- We say that an institution z is empty given s if $N(s, z) = \emptyset$, and nonempty otherwise. The set of all nonempty institutions given s is denoted by $A(s)$.

Biased Institutions - idea

- At every institution z for commodity k where a trader is active, he wants to trade $x_k^i\left(p^i\right)$.
- There are, however, institutions where one market side is rationed. If e.g. buyers are rationed, they can realize only a fraction of their intended trades, whereas sellers face no restriction.
- For each good k there is one fully competitive, Walrasian institution $w_k \in Z_k$ such that no rationing takes place.

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- Commodity 0 is used as the medium of exchange at all institutions for all other commodities and, therefore, there is no rationing for this commodity.
- In order to close the model, the residual trade is conducted with the numeraire good on its market clearing institution $z = 0$.
- This idea of a numeraire good which is traded without rationing is taken from Dreze (1975).

Biased Institutions - model

Institution z is characterized by a rationing parameter $r_z > 0$.

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$$\widetilde{x}_z^i(p,s) = \begin{cases} r_z \cdot x_k^i\left(p^i\right) & \text{if} \quad r_z \leq 1 \text{ and } x_k^i\left(p^i\right) \geq 0 \\ x_k^i\left(p^i\right) & \text{if} \quad r_z \leq 1 \text{ and } x_k^i\left(p^i\right) \leq 0 \\ \frac{1}{r_z} \cdot x_k^i\left(p^i\right) & \text{if} \quad r_z \geq 1 \text{ and } x_k^i\left(p^i\right) \leq 0 \\ x_k^i\left(p^i\right) & \text{if} \quad r_z \geq 1 \text{ and } x_k^i\left(p^i\right) \geq 0 \end{cases}$$

where $z = z(s,i,k) \in Z_k$ is such that $i \in N(s,z)$ and $p_k^i = p_{z(s,i,k)}$. The realized excess demand for the numeraire is given by

$$\widetilde{x}_0^i(p,s) = -\sum_{k=1}^{K} p_{z(s,i,k)} \widetilde{x}_{z(s,i,k)}^i(p,s).$$

Note: $r_{w_k} = 1$ for all k, $r_z \neq 1$ for all $z \in Z_k \setminus \{w_k\}$.

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(i) $\sum_{i \in N(s,z)} \widetilde{x}^i_z(p^*, s) = 0$,

(ii) $\sum_{i=1,\ldots N} \widetilde{x}^i_0(p^*, s) = 0$.

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Lemma. *Assume A0. For every $r = (r_z)_{z \in Z}$ and $s \in S$, there exists a unique (r, s)-equilibrium with strictly positive equilibrium prices at every nonempty institution.*

...hence institution choice yields a well-defined game.

The Learning Model - Intuition

- An example of the learning models we have in mind:
  Each trader compares the currently observed outcomes at all the nonempty trading institutions and switch with positive probability to those yielding the best outcomes, according to the own utility function.

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- Such a rule is myopic in two respects.
  - First, agents do not take into account the fact that switching from one institution to another affects the market outcome.
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  - First, agents do not take into account the fact that switching from one institution to another affects the market outcome.
  - Second, in making such simple, virtual utility comparisons, agents neglect the feedback effects that changes in the market outcome for one good has in the outcome for other goods.
- This is just an example. We will allow for any rule satisfying some minimal behavioral assumptions.

Behavioral Rules (1)

- Institution choice through behavioral rules.
- $B^i : S \rightarrow \Delta S^i$
  i.e. given that the current strategy profile is given by s', $B^i(s')(s^i)$ denotes the probability that trader i will choose the combination of institutions prescribed in s^i next period, for any arbitrary s^i.

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- Given an institution $z \in Z_k$, denote further by $B^i_k(s')(z)$ the probability that trader i will choose institution z for good k next period (marginal probability).

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- Traders might correlate institution choices for different goods.
- Given an institution $z \in Z_k$, denote further by $B^i_k(s')(z)$ the probability that trader i will choose institution z for good k next period (marginal probability).
- We assume, when taking a decision, traders only take their previous decision, prices and rationing of nonempty institutions into account.
- That is, for every nonempty institution, they observe (or care for) only the price and the rationing parameter.
- That is, given $I(s) = \left[A(s), (p_z(s), r_z)_{z \in A(s)}\right]$, we assume that $B^i(s_1) = B^i(s_2)$ whenever $s^i_1 = s^i_2$ and $I(s_1) = I(s_2)$.

Behavioral Rules (2)

- Given a profile s, we say that trader i might leave institution $z \in Z_k$ if $i \in N(s, z)$ but $B_k^i(s)(z) < 1$,
  i.e. the probability that agent i leaves institution z is strictly positive.

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  i.e. the probability that agent i leaves institution z is strictly positive.
- Behavioral Assumption:
  (A1) For every strategy profile s, every good k, every institution $z \in A(s) \cap Z_k$, and every trader $i \in N(s, z)$, trader i might leave z if

  (i) $\widetilde{x}^i_k(s) \geq 0$ and there exists $z' \in Z_k$ with $r_{z'} \geq r_z$ (i.e. buyers are more rationed at z than at z', if at all rationed) and $p_{z'} \leq p_z$, or

  (ii) $\widetilde{x}^i_k(s) \leq 0$ and there exists $z' \in Z_k$ with $r_z \leq r_{z'}$ (i.e. sellers are more rationed at z than at z', if at all rationed) and $p_{z'} \geq p_z$.
- Intuition: a buyer at a given institution z observes that buyers at another institution z' are less rationed and pay a strictly lower price. A myopic buyer will not expect to become a seller if he switches to z' (by A0). A1 states that the buyer wants to switch either to z' or to some other (maybe even better) institution, with at least some probability.

Behavioral Rules (3)

- Extend the reasoning in A1 to institutions where the trader is *not too much rationed*.
- Take a situation where a trader at institution z is not rationed. He observes another institution z' where he would have received a better price, but at the cost of some rationing. Provided that the rationing is moderate compared to the price difference, he wants to switch.

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- **(A1*)** For every strategy profile s, every good k, and every institution $z \in A(s) \cap Z_k$:
  (i) Take $p' < p$. Then there exists a $\underline{r}(p', p) < 1$ such that: if $i \in N(s, z)$ with $\widetilde{x}^i_k(s) \geq 0, p_z = p$, $r_z \geq 1$ and there exists $z' \in Z_k$ with $p_{z'} \leq p'$ and $r_{z'} \geq \underline{r}(p', p)$, then i might leave z.
  (ii) Take $p' > p$. Then there exists a $\overline{r}(p', p) > 1$ such that: if $i \in N(s, z)$ with $\widetilde{x}^i_k(s) \leq 0, p_z = p$, $r_z \leq 1$ and there exists $z' \in Z_k$ with $p_{z'} \geq p'$ and $r_{z'} \leq \underline{r}(p', p)$, then i might leave z.

Behavioral Rules (4)

Lemma (1). *Assume A1. Given any strategy profile $s \in S$ such that, for a good k, both the institution w_k and another, not fully competitive $z \in Z_k$ are nonempty, there either all buyers or all sellers in $N(s, z)$ might leave z.*

Behavioral Rules (4)

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Proof. Suppose $r_z < 1$. This implies that, at z, (weak) sellers are not rationed.

If sellers want to leave institution z, the proof is completed.

If some seller wants to stay at institution z with certainty, then by A1(ii) it follows that $p_{w_k} < p_z$.

Since buyers are rationed at institution z but there is no rationing at w_k, A1(i) implies that all (weak) buyers have positive probability to leave institution z.

The proof for $r_z > 1$ is analogous. ■

Behavioral Rules (5)

- **(A2)** For every strategy profile s, every good k, and every two institutions $z, z' \in Z_k$, we have that, if z is nonempty and z' is empty under s, then

$$B_k^j(s)(z') = 0 \text{ for all } j \in N(s, z).$$

- Intuition: Traders prefer trading over no trading. Hence they never switch to empty institutions.
- Alternatively, this assumption can be interpreted as an information constraint: empty institutions are not even observed, hence they are not perceived as alternatives.

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Revision opportunities

- State-dependent, random revision opportunities.
  Let $E(i, s)$ denote the event that agent i receives revision opportunity when the current state is s, and let $E^*(i, s) \subseteq E(i, s)$ denote the event that agent s is the only agent receiving revision opportunity in s.
  $E(i, s)$: event that trader i receives revision opportunity at profile s.
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  $E^*(i, s) \subseteq E(i, s)$: event that i is the only trader with revision opportunity at s.
- Assumption **(D)**: $\Pr\left(E^*(i, s)\right) > 0$ for every agent i and state s.
  Implies that $\Pr\left(E(i, s)\right) > 0$, i.e. every agent has strictly positive probability of being able to revise at any given state.

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  Implies that $\Pr(E(i, s)) > 0$, i.e. every agent has strictly positive probability of being able to revise at any given state.
- Encompasses many standard learning models, like those with
  \- *independent inertia*: Exogenous, independent, strictly positive probability, that an agent does not revise.
  \- *non-simultaneous learning*: only one agent per period has positive probability of revision.
- $\Pr(E(i, s))$ might also depend e.g. on the difference of payoffs between different institutions (so that unsatisfied traders are more likely to revise), or on idiosyncratic characteristics of the currently chosen institution.

Mistakes / Experiments

- (D) + B_i's yield a Markov Chain on the (finite) state space S.
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- (D) + B_i's yield a Markov Chain on the (finite) state space S.
- Multiplicity of absorbing sets/states, e.g. full coordination on any institution combination.
- Stability check: small experimentation probability $\varepsilon > 0$.
  (the "mistakes model" of KMR93, Young 93, etc...)
- In case of experimentation: institution chosen at random, prob. distribution with full support over institutions.
- Unique invariant distribution $\mu(\varepsilon)$ with full support on S.
- limit invariant distribution $\mu^* = \lim_{\varepsilon \to \infty} \mu(\varepsilon)$
- Stochastically stable states: those in the support of μ^*.
- Techniques in the proofs: Ellison (2000).

Stability of Walrasian institutions

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Theorem. *Under A1, A2, and D, W is stochastically stable.*

Rough idea of the proof:

Transitions towards W happen with high probability (few simultaneous mutations).

From any state where $z \neq w_k$ is nonempty, a single mutation puts one trader in w_k. By Lemma 1 and Assumption D, some trader leaves z. Repetition of this argument empties z.

Iteration empties all institutions for good k other than w_k (by A2, none of the empty institutions can become nonempty in the process).

Iteration over goods leads to state W.

Apply Radius/Modified Coradius Theorem in Ellison (00).

Stability of other institutions

Observation: The equilibrium price vector varies continuously with the rationing parameters.

Theorem. *Assume A1,A1*,A2, and D. For generic economies, there exist* $\underline{r}_k < 1$ *and* $\overline{r}_k > 1$ *for all* k *such that, if* $z_k(r_k)$ *is an institution for good* k *with rationing parameter* $r_k \in]\underline{r}_k, \overline{r}_k[$*, the state* ω *where all traders coordinate at the institutions* $z_k(r_k)$ *for all* k *is stochastically stable.*

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Very rough idea of the proof:

It is relatively simple to reach ω from W.

Start at W and take good 1.

By a continuity argument and A1*, we can find r_1 close enough to 1, there always exists a trader at w_1 who wants to change to $z_1(r_1)$, if only those two institutions for good 1 are nonempty (use A2).

Genericity is needed to avoid price ties among equivalent institutions holding different sets of traders.

Conclusion

- Coordination on the market-clearing institutions is obtained independently of the characteristics of the alternative available trading institutions.
- This strong stability result shows that the market-clearing “assumption” is justified, to a certain extent.
- On the other hand, some alternative non market-clearing institutions are also stochastically stable.
- Nothing guarantees that the actually used trading institutions are efficient - some regulatory interventions might be necessary to improve the functioning of trading institutions.
- Furthermore, non-market clearing “stable” institutions can be deliberately designed.